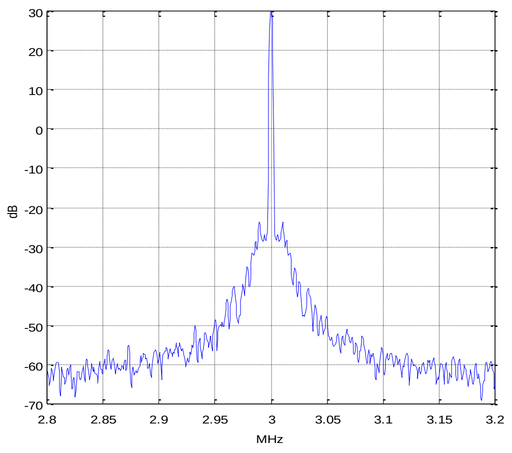
ADC Clock Jitter Model, Part 2 – Random Jitter

In Part 1, I presented a Matlab function to model an ADC with jitter on the sample clock, and applied it to examples with deterministic jitter. Now we'll investigate an ADC with random clock jitter, by using a filtered or unfiltered Gaussian sequence as the jitter source. What we are calling jitter can also be called time jitter, phase jitter, or phase noise. It's all the same phenomenon. Typically, we call it jitter when we have a time-domain representation, and noise when we're in the frequency domain.

We'll look at three examples: band-limited Gaussian jitter, wideband Gaussian jitter, and close-in phase noise. In all of the examples, the analog input signals are pure sinewaves, either single or multiple. So keep in mind that the noise on the output signals is caused by the sample clock (and to a lesser degree by quantization). A Matlab function `adc_jitter` for modeling the ADC jitter is listed in the appendix of Part 1. I've also added a modified version, `adc_jitter_cubic` in Appendix B of this article.



Example 1. Band-limited Gaussian jitter

In this example, we model an ADC with band-limited Gaussian jitter on its sample clock. We use a simple Butterworth filter to provide the band limiting. While this is not a typical real-world case, it shows that random jitter sidebands behave similarly to the sinusoidal jitter we modeled in Part 1.

As in Part 1, we have to create the input signal and the time jitter vector before we call the Matlab function. First, we define the ADC sample rate and input signal frequency. Also, we define the simulation sample rate, which is twice the ADC sample rate.

```
randn('state',1)        %  reset random number generator
fs_adc= 10E6;           % Hz ADC sample rate
f0 = 1E6;               % Hz ADC input sinewave frequency
fs= 2*fs_adc;           % Hz simulation sample rate
```

Next, define the analog input sinewave x:

```
Ts= 1/fs;
N= 2^15;
n= 0:N-1;

x= sin(2*pi*f0*n*Ts);               % adc input
```

Now we create a lowpass Butterworth filter and use it to filter a Gaussian random sequence. The 7th order filter's cutoff frequency is 0.5 MHz. The units of the jitter amplitude is seconds. We compute the jitter per sample = dt/Ts, then call the Matlab function `adc_jitter`. Finally, we quantize the output to 10 bits.

```
% create random band-limited jitter

fc= .5e6;                        % Hz cutoff freq of noise filter
[b,a]= butter(7,2*fc/fs);        % coeffs of noise filter

A= 1e-9;                         % s jitter std. deviation before filtering
u= A*randn(1,N);                 %  AWGN sequence

dt= filter(b,a,u);            % s   filtered sequence = jitter of sample clock
dsample= dt/Ts;               %     jitter per sample


y= adc_jitter(x,dsample);              % ADC jitter function

nbits= 10;             %    ADC number of bits (ideal quantization)
y= floor(2^(nbits-1)*y)/2^(nbits-1);
```

The output y is at the fs_adc sample rate of 10 MHz. Figure 1 shows the spectra of the input sine wave and ADC output. The jitter has created a noise pedestal with bandwidth of 1 MHz. Now let's create an input signal using two sine waves, one centered at 1 MHz, and the other at 4 MHz:

```
x= .5*(sin(2*pi*f0*n*Ts) + sin(2*pi*4*f0*n*Ts));       % adc input
```

The resulting spectra are shown in Figure 2. As you can see, the noise pedestal of the high frequency signal is about 12 dB higher than that of the low frequency signal. This agrees with the relationship of Equation 3 in Part 1 for sinusoidal jitter, i.e., sideband level varies as $20*\log_{10}$ of the input signal's carrier frequency: $20*\log_{10}(4\text{ MHz}/1\text{ MHz}) = 12$ dB.

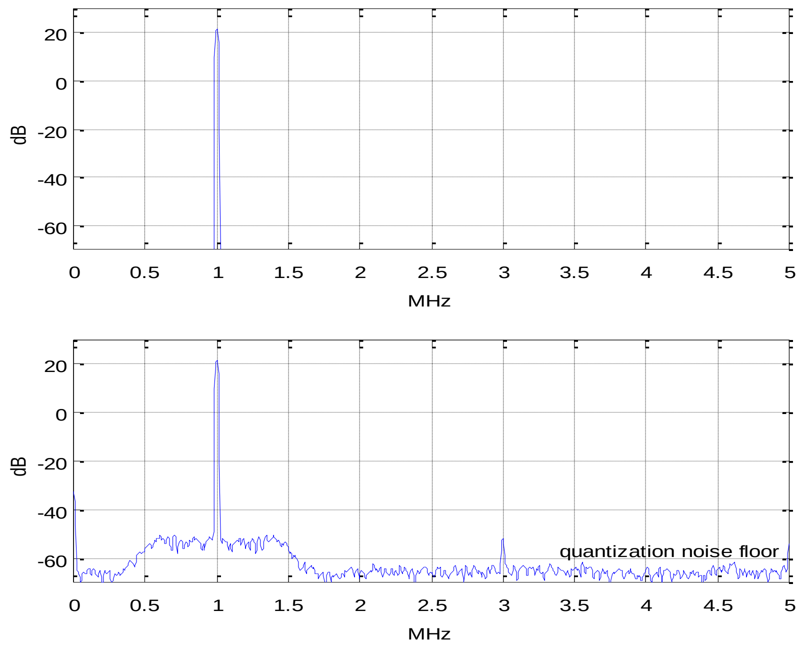


Figure 1. ADC with band-limited Gaussian jitter on sample clock and sinusoidal input.

Top: Spectrum of input signal Bottom: Spectrum of output signal

```
Nadc= N/2;  psd(y,Nadc/8,fs_adc/1e6,flattopwin(Nadc/8))
```

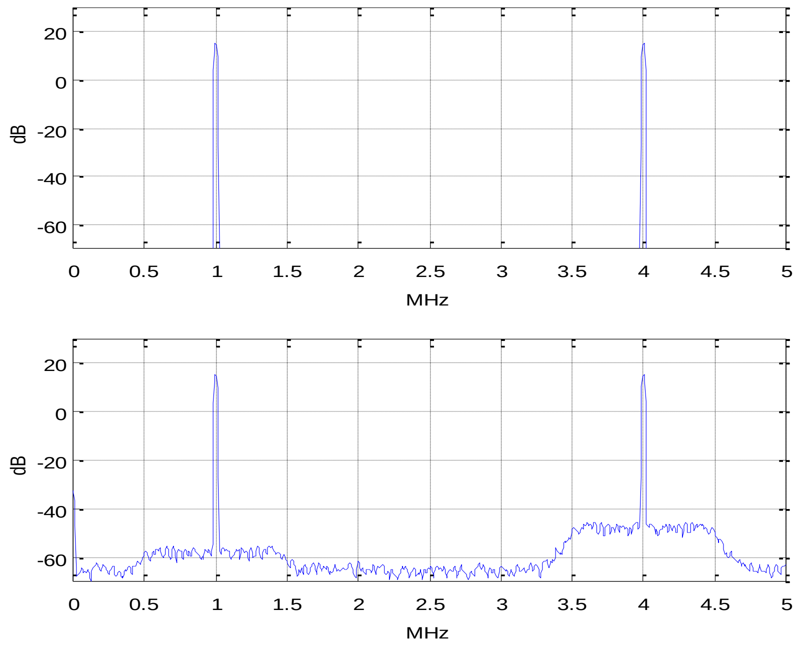


Figure 2. ADC with band-limited Gaussian jitter on sample clock and dual sinusoidal input.
Top: Spectrum of input signal Bottom: Spectrum of output signal

Example 2. Wideband Gaussian jitter

Figures 3 and 4 show the spectra for a sine input to the ADC at 4 Mhz. Both have the same standard deviation of the sample clock jitter to start, but in Figure 3, the jitter is filtered, while it is not filtered in Figure 4 (wideband jitter). The wideband jitter is generated as follows, using the same standard deviation as example 1:

```
A= 1e-9;                      % s jitter std. deviation
u= A*randn(1,N);              %  AWGN sequence
dt= u;                        % s  jitter of sample clock
```

We might expect the noise levels near 4 MHz to be the same, but the noise of the wideband jitter case is about 3 dB higher. The reason is that when `adc_jitter` downsamples the signal by 2, the wideband noise between $f_s/2$ and f_s is aliased into the 0 to $f_s/2$ band. We can call this phenomenon a "feature" of `adc_jitter`, rather than a bug, because it occurs in real-world ADC's as well. In fact, the noise energy of the sample clock can alias multiple times, since the bandwidth of the sample clock path is normally several times the sample frequency. Unfortunately, the user does not typically know this bandwidth.

If the wideband noise floor is too high, it can harm the ADC's SNR more than the close-in phase noise. However, low-pass filtering the sample clock is not a cure-all; if the edge is too slow, jitter can be added due to ground noise being transferred to the clock. See [1] for a more detailed discussion.

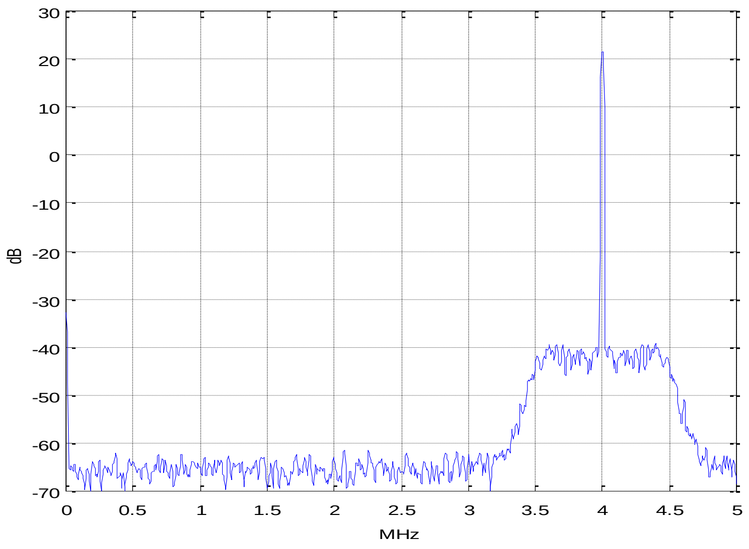


Figure 3. Spectrum of ADC output with band-limited Gaussian jitter on sample clock and sinusoidal input at 4 MHz.

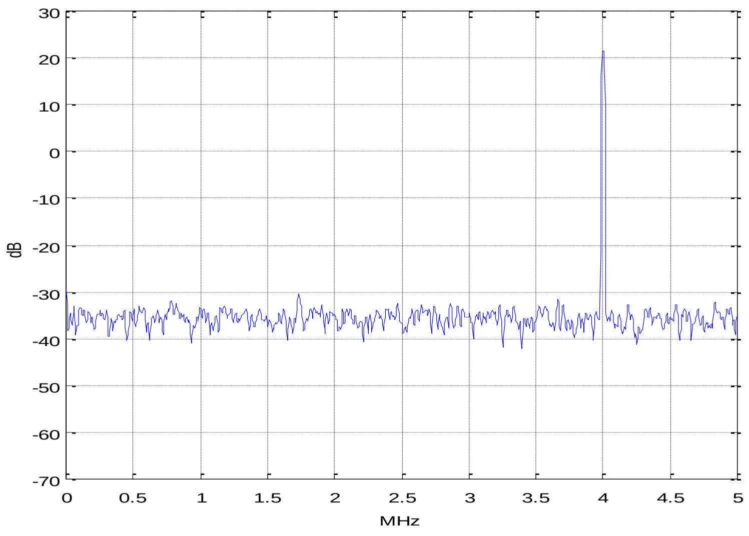


Figure 4. Spectrum of ADC output with wideband Gaussian jitter on sample clock and sinusoidal input at 4 MHz.

Here's another peculiar effect of wideband jitter. As we have seen, the spectral noise due to sample clock jitter depends on the frequency of the input signal. Consider an example shown in Figure 5 with wideband Gaussian noise on the sample clock. There are noiseless input sinewaves at 1 MHz and 2.3 MHz, and the signal at 2.3 MHz is 20 dB lower than the signal at 1 MHz. If we move the larger signal to 4 MHz, the noise floor rises by 12 dB (as in Example 1). So the "victim" signal at 2.3 MHz sees its SNR degraded by 12 dB, just due to the new frequency of the larger signal.

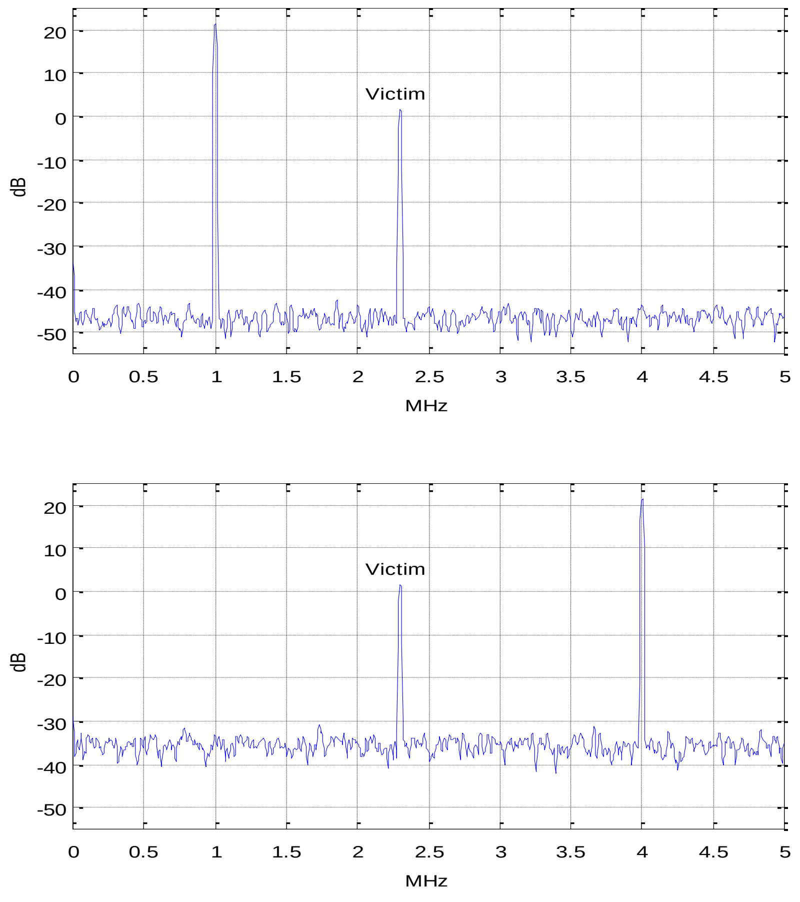


Figure 5. ADC output spectra for two frequencies of input aggressor signal.
Top: Aggressor signal at 1 MHz. Bottom: Aggressor signal at 4 MHz.

Example 3. Close-in Phase Noise

In this example, we model sample clock close-in phase noise by shaping Gaussian noise to resemble a real-world clock source [2]. The noise has a flat region up to almost 10 kHz, followed by frequency ranges with -12 dB/octave and -6 dB/octave slope, followed by flat noise. We achieve this by synthesizing a filter with the desired response and using it to filter Gaussian noise. A Matlab function to synthesize the filter is listed in Appendix A.

As in the earlier examples, I started out using parabolic interpolation to model the ADC jitter, but the resulting jitter at the ADC output was typically about 7% less than the sample clock jitter. I switched to cubic interpolation [3, 4] and got more accurate results. The Matlab function `adc_jitter_cubic` is listed in Appendix B (The parabolic interpolation version is listed in the appendix of Part 1). Note that the cubic interpolator frequency response for mu= 0.5 is about 1 dB down at $f_s/4$.

The ADC input is a noiseless sinewave. We define the various simulation parameters as in Example 1, noting that input sinewave frequency f_0= 3 MHz and the number of samples N is 2^17 = 131,072.

```
randn('state',3)        % initial state of RN generator
fs_adc= 10E6;           % Hz ADC sample rate
f0 = 3E6;               % ADC input sinewave frequency

fs= 2*fs_adc;           % Hz simulation sample rate

Ts= 1/fs;
N= 2^17;
n= 0:N-1;

x= sin(2*pi*f0*n*Ts);               % adc input
```

Next we define the noise shaping filter, using three frequencies to determine the regions of different slope. Then we filter the Gaussian sequence and call the function `adc_jitter_cubic`. We quantize the ADC output to 10 bits.

```
f1= 10000; f2= 100000; f3= 1e6;      % Hz filter breakpoints
[b,a]= noise_filter(f1,f2,f3,fs);

A= 9e-9;                             % s jitter sigma before noise filter
u= A*randn(1,N);

dt= filter(b,a,u);                   % s  jitter of sample clock
dsample= dt/Ts;                      %   jitter per sample

y= adc_jitter_cubic(x,dsample);      % ADC jitter function (cubic interp)

nbits= 10;                         % ADC number of bits (ideal quantization)
y= floor(2^(nbits-1)*y)/2^(nbits-1);    % quantize to nbits
```

The resulting spectrum at the ADC output is shown in Figure 6, which has a frequency span of 400 kHz. For an ADC output sequence of length 2^{16}, we employed spectrum averaging with a psd of length of $2^{16/4}$ = 16,384 samples. For our ADC sample frequency of 10 MHz, this gives a frequency resolution of roughly 1 kHz using a flat-top window. Had we used a sample frequency of 100 MHz, the frequency resolution would have been very coarse at about 10 kHz. This shows the challenge of modeling close-in phase noise in the frequency domain: the signal of interest has bandwidth much less than the sample rate. So we need many samples to get the desired frequency resolution.

Now let's compare the jitter on the ADC output signal to the sample clock jitter: they should be approximately equal. To do this, we have to phase-demodulate the signal. The code to perform demodulation is listed in Appendix C. We call the demodulator output φ_d. The time jitter is:

$$\Delta t = \frac{\phi_d}{2\pi} T_0$$

$$= \frac{\phi_d}{2\pi f_0}$$

The jitter on the sample clock and on the ADC output signal are plotted as histograms in Figure 7. We can also compare the rms jitter of the sample clock and ADC output. RMS jitter is:

$$\Delta t_{rms} = \left(\frac{\sum (\Delta t)^2}{N} \right)^{1/2}$$

Here are the Matlab calculations. The rms output jitter is only 1.2% away from the rms sample clock jitter:

```
dt_ps_rms= sqrt(sum(dt_ps.^2)/N)     % ps  rms jitter of sample clock

jitter_ps_rms= sqrt(sum(jitter_ps.^2)/length(jitter_ps))     % ps rms jitter
of output

dt_ps_rms =  276.7063
jitter_ps_rms = 280.0680
```

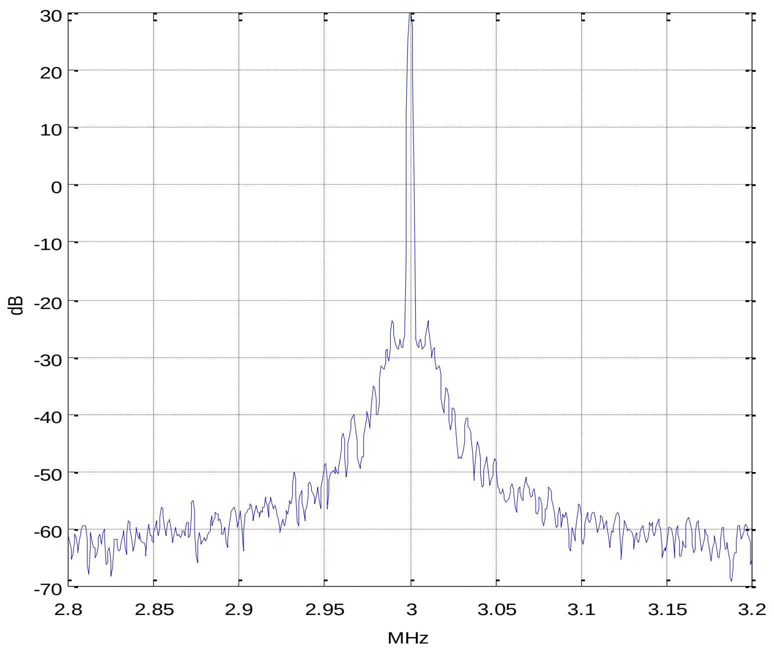


Figure 6. Spectrum of ADC output for sample clock with shaped Gaussian noise.

```
Nadc= N/2;  psd(y,Nadc/4,fs_adc/1e6,flattopwin(Nadc/4))
```

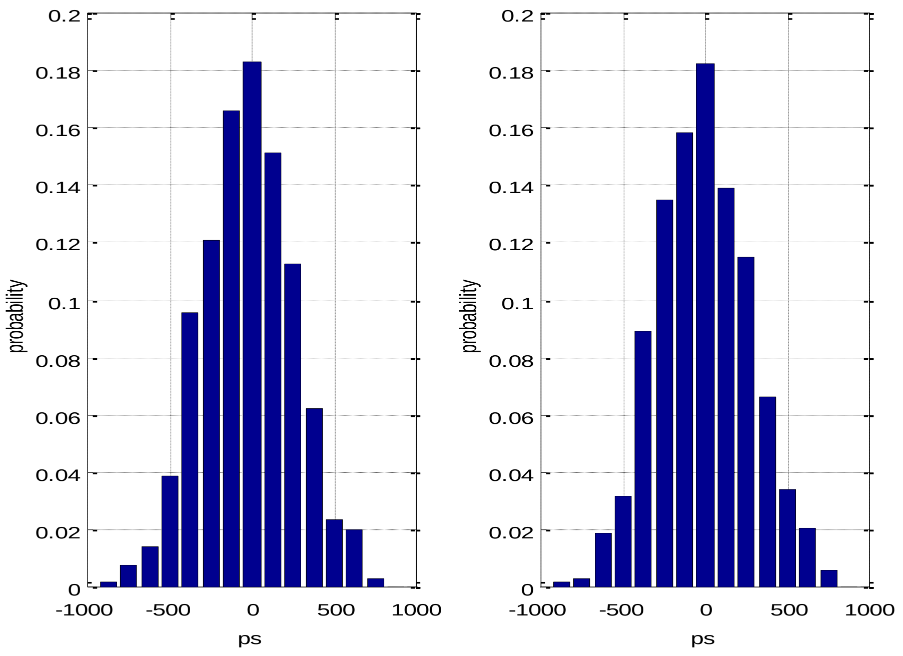


Figure 7. Jitter Histograms for shaped Gaussian noise on sample clock

Left: Jitter of sample clock. Right: Jitter on sinewave at ADC output, f_0 = 3 MHz.

References

1. Brannon, Brad, “Sampled Systems and the Effects of Clock Phase Noise and Jitter”, Analog Devices Application Note AN-756, 2004
http://www.analog.com/media/en/technical-documentation/application-notes/AN-756.pdf

2. Goldberg, Bar-Giora, “Phase Noise Theory and Measurements: A Short Review”, Microwave Journal, Jan 1 2000.
https://www.google.com/search?q=%2C+Phase+Noise+Theory+and+Measurements%3A+A+Short+Review&oq=%2C+Phase+Noise+Theory+and+Measurements%3A+A+Short+Review&aqs=chrome..69i57.4669j0j8&sourceid=chrome&ie=UTF-8

3. Erup, Lars; Gardner, Floyd M. and Harris, Robert A., “Interpolation in Digital Modems – Part II: Implementation and Performance”, IEEE Transactions on Communications, Vol 41, No. 6, June 1993.

4. Rice, Michael, Digital Communications, a Discrete-Time Approach, Pearson Prentice Hall, 2009, section 8.4.2.

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Appendix A Matlab Function `noise_filter`

This function synthesizes a noise-shaping filter as a cascade of a 2nd order Butterworth lowpass and two proportional + derivative (PD) filters. The function's inputs are sample rate and three frequencies that set the corner frequencies of the constituent filters. For widely spaced frequencies, the approximate slope of the filter response is:

- flat to f_1
- -12 dB/octave between f_1 and f_2
- -6 dB/octave between f_2 and f_3
- flat above f_3

Each PD filter has flat response from dc to just below the corner frequency f_2 or f_3, then the response rises at +6 dB/octave. H(z) is given by:

$$H(z) = b_0 + b_1 z^{-1} \ ,$$

where, for example, to achieve corner frequency of approximately f_2, b_0 and b_1 are:

$$b_0 = \frac{f_s}{2\pi f_2}$$

$$b_1 = 1 - \frac{f_s}{2\pi f_2}$$

The overall filter response used in Example 3 is shown in Figure A.1 vs. a log frequency scale.

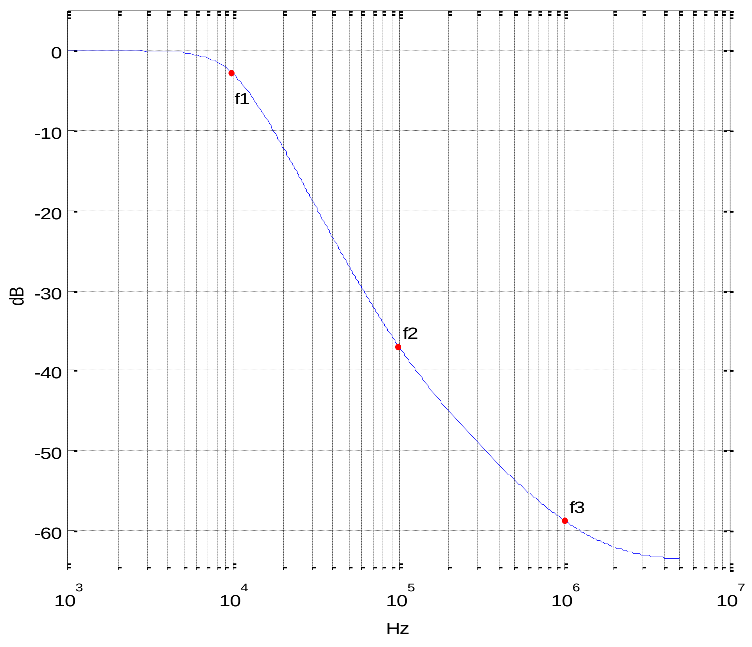

Figure A.1 Noise shaping filter response for f_1 = 10 kHz, f_2= 100 kHz, f_3= 1 MHz, f_s= 10 MHz

```
%[b,a]= noise_filter(f1,f2,f3,fs)  3/6/18 nr
%
% find coeffs of three-filter cascade for noise shaping:
%  2nd order LP, plus two Proportional-Derivative (PD)
% f1 = corner of LP, Hz
% f2= corner of 1st PD filter, Hz
% f3= corner of 2nd PD filter, Hz
% fs= sample frequency, Hz
% for widely spaced frequencies, the approximate slopes are:
%  -12 dB/octave between f1 and f2
%  -6 dB/octave between f2 and f3
%   flat above f3
%
function [b,a]= noise_filter(f1,f2,f3,fs);


% LPF   Use impulse invariant design to avoid null at fs/2.

w1= 2*pi*f1;
num= 1;
den= [1/w1.^2 sqrt(2)/w1 1];  % s-domain 2nd order Butterworth. [1 sqrt(2) 1]

[b,a]= impinvar(num,den,fs);

% PD filter 1
w2= 2*pi*f2;
d0= fs/w2;
d1= 1 - d0;    % FIR PD

d= [d0 d1];

% PD filter 2
w3= 2*pi*f3;
e0= fs/w3;
e1= 1 - e0;
e= [e0 e1];     % FIR PD


b= conv(b,conv(d,e));      % cascade 3 filters  (only 1st filter has denom)
```

Appendix B. Matlab Function `adc_jitter_cubic`

The function `adc_jitter` presented in Part 1 used parabolic interpolation. This version uses cubic interpolation. Figure B1 shows the frequency response of the cubic interpolator for mu= 0.5. Because the output is downsampled by two, only the frequency range from 0 to fs/4 is used.

```
%function y= adc_jitter_cubic(x,dsample)  4/21/18     Neil Robertson
% Model ADC with jitter on sample clock, computing jittered samples by
% cubic interpolation.
% Add jitter to input signal x, then downsample by 2.
%
% x            input signal vector
% dsample      input jitter vector, jitter in samples
% y            output signal vector with jitter, sample freq = 1/2 of input fs
%
function y= adc_jitter_cubic(x,dsample)

if length(x)~=length(dsample)
   error('   x and dsample must be of equal length')
end

N= length(x);
V= x;

% find jittered samples using cubic interpolation

mu= 0.5 + dsample;          % mu = 0.5 +/- jitter

% Farrow coefficients
b1= [-1 6 -3 -2]/6;
b2= [0 1 -2 1]/2;
b3= [1 -3 3 -1]/6;

u= zeros(1,N);
for n= 4:N;
   Vreg= V(n:-1:n-3);       % reg holding 4 samples of V, current sample first
   u(n)= Vreg(3) +...
      mu(n)*( sum(b1.*Vreg) + mu(n)*(sum(b2.*Vreg) + mu(n)*sum(b3.*Vreg)));
end

y= u(1:2:end);              % downsample by 2
```

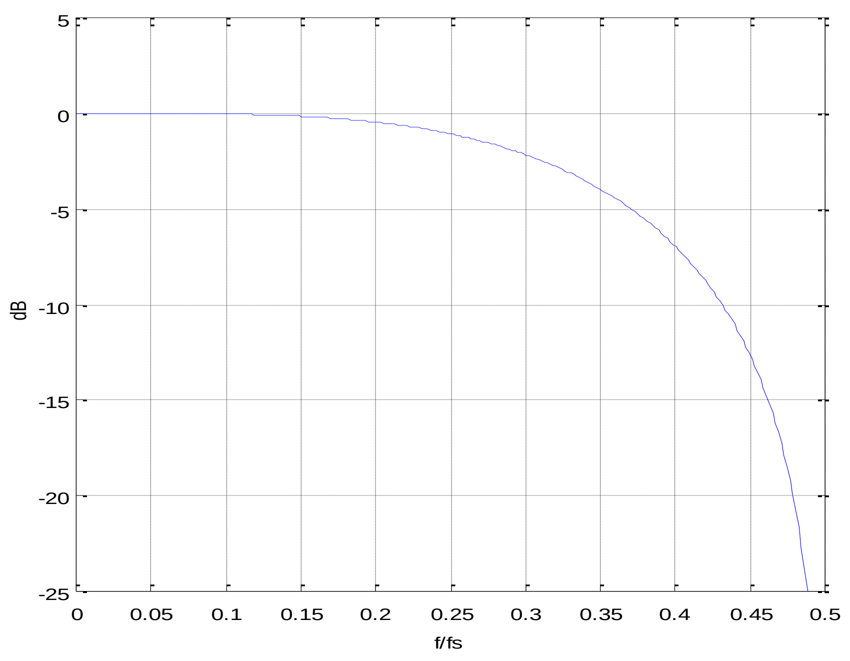

Figure B.1 Cubic Interpolator frequency response for mu= 0.5.

Appendix C. Matlab Code to Demodulate Phase of ADC Output

Here is the code used to demodulate the ADC output in Example 3 and plot the jitter histograms in Figure 7.

```
% demodulate ADC output y

Ts_adc= 1/fs_adc;
n= 0:Nadc-1;

LO= exp(-j*2*pi*f0*n*Ts_adc);     % complex local oscillator at f0

d= LO.*y;                         % complex output of mixer

fc= .5E6;
[b,a]= butter(5,2*fc/fs_adc);     % demodulator LPF coeffs


z= filter(b,a,d);                 % perform LP filtering

I= 2*real(z(65:end));
Q= 2*imag(z(65:end));

phi_d= atan(Q./I);                 % demodulated phase
mean= sum(phi_d)/length(phi_d);    % mean phase
phi_d= phi_d - mean;               % center phase at 0 radians


ps= -1000:125:1000;               % ps histogram bins

% plot sample clock jitter dt_ps as a histogram
[m,ps]= hist(dt_ps,ps);
p= m/sum(m);                      % probability of each bin
subplot(121),bar(ps,p),grid
xlabel('ps'),ylabel('probability'), axis([-1000 1000 0 .2])

% plot ADC output jitter (jitter_ps) as a histogram

jitter_ps= 1E12* phi_d/(2*pi*f0);
[m,ps]= hist(jitter_ps,ps);
p= m/sum(m);                      % probability of each bin
subplot(122),bar(ps,p),grid
xlabel('ps'),ylabel('probability'),axis([-1000 1000 0 .2])
```