Decimator Image Response

This article presents a way to plot the image response of a decimator. A decimate-by-4 filter with coeffiecients b is shown in Figure 1. The filter is followed by a downsampler with output sample rate $f_{s\_out} = f_s/4$. Our goal is to plot the passband response and image response at the output of the downsampler.

Referring to Figure 1, the passband response for f = 0 to $f_{s\_out}/2$ (0 to 0.5 Hz) is just the spectrum of b_down. Figure 2 shows a test source for obtaining the image response. The idea is to apply a flat test spectrum over the frequency $f_{s\_out}/2$ to $f_s/2$ (0.5 to 2 Hz). Since such an input has no energy between f = 0 to $f_{s\_out}/2$, the resulting spectrum of the output will be the image response. The test spectrum is obtained by using a brick-wall hpf with corner frequency $f_{s\_out}/2$. The HPF stopband must not contribute to the image response, thus its stopband must be much lower than the stopband of the decimation filter.

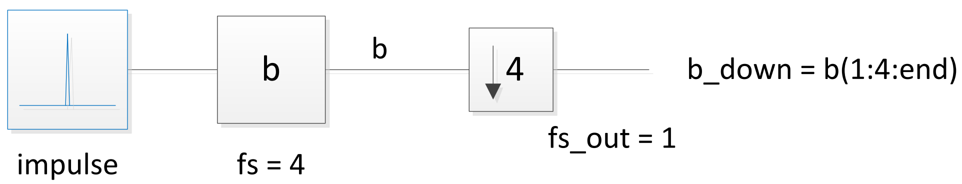

Figure 1. Decimate-by-4 Filter with coefficients b.

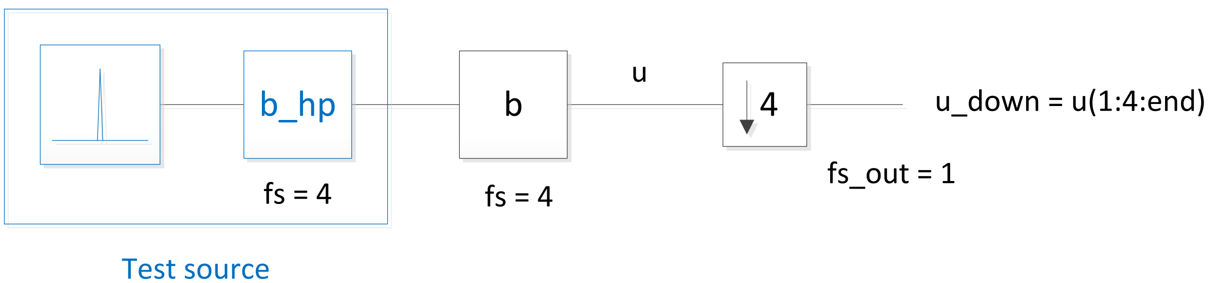

Figure 2. Test source to obtain the image response of the Decimate-by-4 filter b.

The coefficients of the example decimation filter are as follows:

```
b= [1 0 -3 -6 -7 0 15 27 25 0 -45 -80 -74 0 144 315 456 512 456 315 144 0 …
    -74 -80 -45 0 25 27 15 0 -7 -6 -3 0 1]/2048;
```

Compute the frequency response at the input sample rate f_s (See figure 3, Top).

```
fs = 4;                        % input sample rate
[h,f]= freqz(b,1,512,fs);      % frequency response of lpf at fs
H= 20*log10(abs(h));
```

Compute the passband frequency response at the ouput sample rate $f_{s\_out} = f_s/4$.

```
b_down= 4*b(1:4:end);                % impulse response after downsampling
[h,f]= freqz(b_down,1,512,fs/4);     %freq response at fs/4 after downsampling
H1 = 20*log10(abs(h));
```

Create a brick-wall HPF with corner frequency = $f_{s\_out}/2$ = 0.5 Hz (See Figure 3, Bottom).

```
f = [0 .45 .55 2];         % Hz stopband and passband frequencies for fs = 4
f = f/2;                   %    frequencies relative to fs/2
a = [0 0 1 1];             %    stopband and passband response goals

b_hp = firpm(180,f,a);     %    calculate hp filter coeffs N= 180

[h,f]= freqz(b_hp,1,1024,fs);   % hpf frequency response at fs
H= 20*log10(abs(h));
```

Find the impulse response of the cascade of the HPF and the decimation filter, then downsample. Find the frequency response of the downsampled impulse response.

```
u = conv(b,b_hp);             % impulse response at lpf output
u_down = u(1:4:end)*4;        % downsample


[h,f]= freqz(u_down,1,512,fs/4);     %freq response at fs/4 after downsampling
H2= 20*log10(abs(h));
```

The passband frequency response H1 and the image response H2 are plotted in Figure 4.

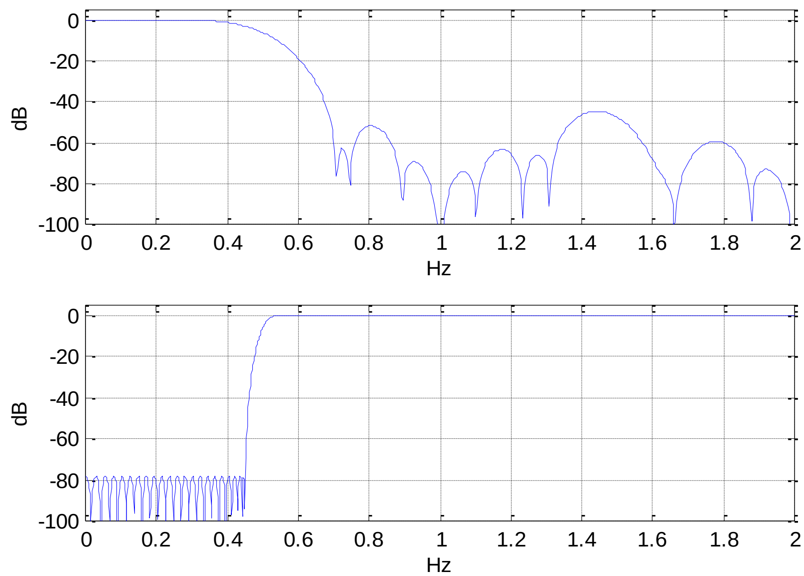

Figure 3. Top: response of decimation LPF at f_s = 4 Hz

Bottom: response of brick-wall hpf with corner freq = $f_{s\_out}/2$ = 0.5 Hz

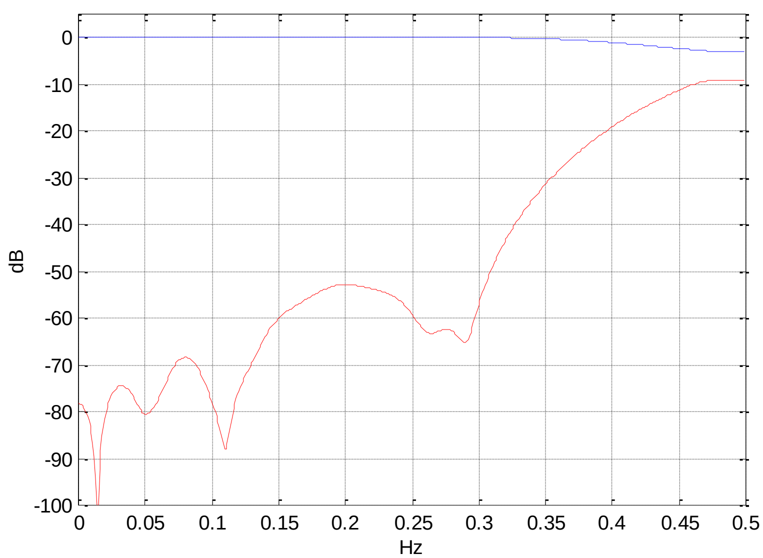

Figure 4. Decimator passband response (blue) and image response (red) at $f_{s\_out} = f_s/4$.

```
% decimator_plot1.m      4/2/16 nr
% plot images for decimate-by-4
% 1.  Find response from 0 to fs in usual way.
% 2.  Create a brick-wall hpf with fc = fs/8.  The spectrum of the
%     impulse response of this filter is in the image band.
% 3.  Convolve hpf with the decimator coeffs and downsample to get
%     impulse response.  Take freqz of the impulse response to find
%     the image spectrum.

fs = 4;          % input sample rate

% 1. decimate-by-4 lpf
b= [1 0 -3 -6 -7 0 15 27 25 0 -45 -80 -74 0 144 315 456 512 456 315 144 0 …
   -74 -80 -45 0 25 27 15 0 -7 -6 -3 0 1]/2048;

[h,f]= freqz(b,1,512,fs);         % frequency response of lpf at fs
H= 20*log10(abs(h));

subplot(211),plot(f,H),grid
axis([0 fs/2 -100 5])
xlabel('Hz'),ylabel('dB')

b_down= 4*b(1:4:end);               % impulse response after downsampling
[h,f]= freqz(b_down,1,512,fs/4);    %freq response at fs/4 after downsampling
H1 = 20*log10(abs(h));


% 2. Brick-wall HPF to create filtered impulse at input

f = [0 .45 .55 2];          % Hz stopband and passband frequencies for fs = 4
f = f/2;                    %    frequencies relative to fs/2
a = [0 0 1 1];              %    stopband and passband response goals

b_hp = firpm(180,f,a);      %    calculate hp filter coeffs

[h,f]= freqz(b_hp,1,1024,fs);  % hpf frequency response at fs
H= 20*log10(abs(h));

subplot(212),plot(f,H),grid
axis([0 fs/2 -100 5])
xlabel('Hz'),ylabel('dB'),figure

% 3.  impulse response with hpf

u = conv(b,b_hp);            % impulse response at lpf output
u_down = u(1:4:end)*4;       % downsample


[h,f]= freqz(u_down,1,512,fs/4);   %freq response at fs/4 after downsampling
H2= 20*log10(abs(h));
plot(f,H1,f,H2,'r'),grid
axis([0 fs/8 -100 5])
xlabel('Hz'),ylabel('dB')
```

4/2/16